USE OF PRODUCED WATER FOR OIL AND GAS DRILLING IN THE SAN JUAN BASIN

Daniel W. Erskine, Maxim Technologies, Inc., Albuquerque, NM; Patrick W. Bergman, and Dave L. Wacker, Conoco E&P Americas, Houston, TX

ABSTRACT

In an effort to improve cost effectiveness and to reduce the use of fresh water, Conoco Inc. (Conoco) has studied the possibility of using produced water to build mud for drilling projects in New Mexico's San Juan Basin. Detailed analysis reveals that it is unlikely that use of produced water will negatively impact human health or the environment for the following reasons:

- Good quality water occurs only locally in the San Juan Basin.

- When produced water is used to build drilling mud, osmotic pressure acts to stabilize the mud /groundwater interface.

- Chemical interactions further stabilize the mud/groundwater interface.

- The combination of advection and dilution will reduce a chloride concentration of 10,000 parts per million (ppm) to less than one ppm in a year's time, over a distance of 50 meters.

- There is enough water within approximately 15 feet of the drill hole to dilute a concentration of 10,000 ppm chloride down to the Federal Drinking Water Standard of 250 ppm.

INTRODUCTION

In an effort to improve cost effectiveness and to reduce the use of fresh water, Conoco is exploring the possibility of using produced water to build mud for drilling projects in New Mexico's San Juan Basin. Several other companies in the area are believed to be using produced water for drilling activities. The State of New Mexico has indicated that only verbal notification would be required to use produced water in drilling activities. The Bureau of Land Management (BLM) requires a beneficial use permit, which has been prepared and reviewed by the agency. The BLM expressed concerns about using a more saline fluid to drill through shallow, fresh water sands. Subsequently, Conoco requested that Maxim Technologies, Inc. (Maxim) prepare this white paper discussing issues relating to BLM concerns; specifically, the possibility that water wells near the drilling could be adversely affected by salt concentrations from the drilling mud. The paper examines forces acting on groundwater, soil moisture, and drilling mud built using produced water and concludes that the use of produced water will not impact human health or the environment

GEOLOGY

The San Juan Basin is a Laramide (Late Cretaceous-Early Tertiary) depression lying at the eastern edge of the Colorado Plateau. Maximum structural relief is reported to be in excess of 10,000 feet, recorded in an oil well near the structural center of the basin (Figure 1). Sedimentary rocks of Jurassic and Cretaceous age crop out around the basin rim and over broad areas in the southern and western parts of the basin. Tertiary sedimentary rocks cover most of the central basin (northeast part of the area). Quaternary deposits are restricted mainly to major valleys.

Groundwater in the San Juan Basin is obtained from Quaternary valley-fill deposits and from sandstones of Tertiary, Cretaceous, Jurassic, and Triassic age. The Glorieta Sandstone and San Andres Limestone (Permian) combine to form a significant aquifer along the southern margin of the basin. Older strata are too deep or too poorly known to be used in any but basin-margin areas (1).

Groundwater Movement

In general, regional groundwater flow is from topographically high outcrop areas toward lower outcrop areas. Much of the recharge to aquifers in the New Mexico part of the basin occurs on the flanks of the Zuni, Chuska, and Cebolleta Mountains (1). Also contributing to the regional flow systems is recharge in high areas along the northern and northeastern basin margins, including the San Juan Mountains in Colorado. The San Juan River valley in the northwest part of the basin and tributaries of the Rio Grande, such as the Rio Salado, Rio Puerco, and Rio San Jose in the southeast parts of the basin, are the main discharge areas for the basin. Of lesser importance in terms of the volume of outflow is the Puerco River near Gallup. Steady-state analysis gives inflow and outflow rates of less than 20 ft^3/S for the Tertiary aquifers and approximately 40 ft^3/S for the Cretaceous and Jurassic sandstone aquifers (2).

Numerous ephemeral-stream channels filled with alluvium are the principal sources of groundwater recharge in some areas and the principal locations of discharge in others. The alluvial cover usually conceals evidence of discharge, and white salt or alkali deposits associated with small-yield springs are often the only surface expression of groundwater discharge in these localities (1). In places, however, the entire floor of ephemeral-stream channels is covered by such deposits (3). X-ray-diffraction analysis of samples of the alkali from several sites by the New Mexico Bureau of Mines and Mineral Resources has revealed it to be thenardite (Na_2SO_4). Most discharge to alluvial channels is lost by evapotranspiration, but some water also moves as subsurface flow.

Interaquifer movement of water (leakage) is part of the groundwater flow system in the San Juan Basin. Hydraulic-head differences of 200 feet or more, which commonly exist between aquifers in many parts of the basin, provide the driving mechanism for such movement. The geologic section in Figure 1 shows the probable direction of flow through confining beds.

Units of Concern

Shallow freshwater aquifers in the San Juan Basin occur primarily in valley fill deposits associated with underlying Cretaceous units in the southern half of the basin and in local sandstones interbedded with mudstones in the Eocene San Jose Formation in the northern part of the basin (1). The only widespread shallow aquifer in the basin is the Ojo Alamo Sandstone, which occurs at the base of the Tertiary section. This unit outcrops in a narrow band that strikes northwest/southeast across the center of the basin and dips to the northeast toward the basin center to a maximum depth of 3,645 feet. Specific conductance in the Ojo Alamo Sandstone increases from less than 1,000 µmhos near outcrops to more than 9,000 µmhos downdip. The Ojo Alamo Sandstone is used as a source of domestic and stock water only in a narrow strip bordered on the south by the outcrop and on the north by New Mexico 44 (Figure 2) (1).

Shallow Groundwater Quality

Specific conductance of groundwater in the San Juan Basin ranges from less than 500 µmhos close to outcrops of some of the more transmissive rocks to more than 100,000 µmhos in deeper, less transmissive units (Stone et al., 1983). The following is a quote from a report focusing on the San Juan Basin and prepared for the Navajo Tribal Utility authority and the U.S. Bureau of Reclamation (4). "Much of the ground water in northwestern New Mexico is of poor quality. Almost without exception groundwater contained in the Quaternary and younger Cretaceous rocks is of poor quality. The Cretaceous contains numerous coal beds, and the Quaternary alluvium, having been derived from the underlying Cretaceous, has a similar composition. Groundwater percolating through these strata, therefore, accumulate high sulfate concentrations. The lowermost Cretaceous sandstones (Gallup and Dakota) and the Jurassic contain no such contaminating beds. Therefore, along the margins of the Defiance and Zuni uplifts groundwater in these formations is potable. The quality of groundwater in these aquifers however deteriorates basinward because the residence time for the water increases and minerals are dissolved in increasing concentrations. Groundwater contained in the Dakota-Morrison-Cow Springs aquifer is potable only in the western and southern portions of the: study area. Water contained in the Gallup sandstone is usually potable only in the southwestern part of the basin." Figure 2 shows areas described above where groundwater is considered to be an aquifer.

CLIMATE

The climate is arid to semiarid, with the highest levels of precipitation (20 to 30 inches per year) in the mountainous areas, and the lowest levels in the central part of the basin (8 to 10 inches per year) (1). The bulk of precipitation occurs as intense summer thunderstorms. Summertime temperatures have been known to be as high as 110^{o} F. The average annual pan-evaporation ranges from 46 to 72 inches per year (1), far exceeding annual precipitation.

DRILLING/DRILLING MUD

Mostly composed of bentonite and water, drilling mud is pumped down the center of the hollow drill stem, and emerges again at the surface, carrying the pulverized rock encountered by the drill bit. The mud is necessary to lubricate the drill shaft, cool the hole, and carry away the detritus that the drilling has created.

Drilling mud deposits a filter cake on the wall of the well bore. This wall cake helps protect the formation by retarding the passage of mud filtrate into the formation. The higher the permeability of a formation, the greater its ability to accept and receive mud filtrate. Therefore, the nature of the filter cake has a direct effect on such problems as formation damage, sloughing and caving, tight hole and stuck pipe problems. A drilling mud with a low filtrate water loss will form a thin, tough filter cake. Specially formulated, bentonite mud provides the filtrate control and a thin, tough filter cake needed. Bentonite, polymers and starch are used to control the water loss

Forces Driving Drilling Mud-Formation Interaction

Two fundamental forces drive the transfer of water between drilling mud and the formation: (1) the hydraulic differential between the fluid pressure in the borehole and (2) the pore pressure in the formation, and osmotic pressure.

Hydraulic Differential

Drilling mud is forced down the drill stem at a higher pressure than the pore pressure in the formation in order to stabilize the hole. This forces mud filtrate into the formation until pressure is equalized and a filter cake is established.

Osmotic Pressure

Theoretically, osmosis occurs when a pure solvent (water) and a solution (water with dissolved salts) are separated by a membrane (e.g., a cell wall) permeable only to the solvent. The pure solvent passes through the membrane, diluting the concentration of the solution on the other side. Bentonite clay particles in the drilling mud act as a semipermeable membrane, and like the membrane of a cell wall, is selective about what it allows to pass through and what it prevents from passing. Water is present on both sides of the mud/formation interface "membrane," with each side having a different concentration of dissolved material. The water will pass very easily because of its small molecular size, while larger molecules or dissolved ions will not pass so easily.

Since the water in the less concentrated solution seeks to dilute the more concentrated solution, water will diffuse through the membrane from the lower concentration side to the greater concentration side. A higher concentration of ions or molecules in the drilling mud results in osmotic movement of water from the formation into the mud. Thus, using higher concentration produced water in place of fresh water in drilling mud will tend to draw water out of the formation.

Eventually, osmotic pressure will counter the diffusion process exactly, creating equilibrium in a closed system, promoting stabilization of the mud/formation system. If the system does decay it will be over a period of time that is long relative to groundwater flow. That is, high concentrations of salt in the drilling mud will bleed slowly into the groundwater as it passes the drill hole, resulting in low concentrations of salt in groundwater.

Chemical Interactions

The surfaces of solids, especially clays, have an electrical charge due to isomorphous replacement, broken bonds, and lattice defects (5). The electrical charge is imbalanced and can be satisfied by adsorbing a charged ion. Clays tend to be strong adsorbers, since they have both a high surface area per unit volume and significant electrical charges at the surface. Most clay minerals (including bentonite) have an excess of imbalanced negative charges in the crystal lattice, therefore, favoring the adsorption of and immobilization of cations such as sodium or calcium.

Adsorption of cations by the bentonite in drilling mud is important because it acts to further limit the diffusion of salts from the drilling mud to fresh water and further increases the stability of the mud/formation system. This is because ions must maintain electrical neutrality as they diffuse. For example, in a solution of NaCl, the Cl^- anion cannot diffuse faster than the Na^+ cation and still maintain charge balance in the water.

PLUME MOVEMENT

There are two basic processes acting to transport solutes as a groundwater plume. Molecular diffusion is a chemical process and advection is a mechanical process. Both processes cause dilution, or reduction of constituent concentrations over time and distance from the source.

Molecular Diffusion

Molecular diffusion is the process by which both ionic and molecular species dissolved in water move from areas of higher concentration to areas of lower concentration (i.e., chemical activity). It originates because of mixing caused by random molecular motions due to the thermal kinetic energy in the water. Because of molecular spacing, the coefficient describing this scattering is higher in gases than in liquids and higher in liquids than in solids. The diffusion coefficient in a porous medium is smaller than in pure liquids because collisions with solids hinder diffusion.

Advection

Advection is the transport of dissolved constituents with the flow of groundwater. As a fluid containing high concentrations of dissolved salts flows through a porous medium, it will mix with water containing lower concentrations of dissolved salts. The result will be a dilution of the salt concentration by a process known as dispersion. The mixing that occurs along the streamline of fluid flow is called longitudinal dispersion. Dispersion that occurs normal to the pathway of fluid flow is lateral dispersion.

There are three basic causes of longitudinal dispersion. (A) As fluid moves through pores, it will move faster through the center of the pore than along the edges. (B) Some of the fluid will travel in longer pathways than other fluid. (C) Fluid that travels through larger pores will travel faster than fluid moving in smaller pores. Figure 3 illustrates these three causes. Lateral dispersion is caused by the fact that, as a fluid containing a dissolved salt flows through a porous medium, the flow paths can split and branch out to the side (Figure 4). This will occur even in the laminar flow conditions that are prevalent in groundwater flow.

Dilution

The processes of molecular diffusion and advective dispersion cannot be realistically separated in flowing groundwater. Instead, a factor termed the coefficient of hydrodynamic dispersion takes into account both mechanical mixing and diffusion. For one-dimensional flow, hydrodynamic dispersion is represented by the following equation:

$$D_L = a_L v_x + D^* \quad (1)$$

Where:

D_L = the longitudinal coefficient of dispersion
a_L = the dispersivity
v_x = the average linear groundwater velocity
D^* = the molecular diffusion

The average linear velocity is calculated using the hydraulic conductivity (K), the hydraulic gradient (dh/dl), and the effective porosity (n_e) as follows:

$$v_x = K(dh/dl)/n_e \quad (2)$$

In practice, the value of a_L is estimated as 0.1 times the flow length of interest and D* is estimated to be 1×10^{-9} m^2/sec (6). The resulting estimate of D_L allows the calculation of a concentration (C), at some distance (L), from the source concentration (C_o), at time (t) using the following equation where erfc is the complementary error function:

$$C = C_o/2[\mathrm{erfc}((L-v_x t)/2(D_L t)^{0.5} + \exp(v_x L/D_L) + [\mathrm{erfc}((L+v_x t)/2(D_L t)^{0.5}] \quad (3)$$

To apply this equation to the question of salts from drilling mud affecting shallow aquifers, we can assume an unrealistic case where the factors that cause stability in the drilling mud and require that salts from the drilling mud to bleed off over time do not hold. In this hypothetical case, we assume that the highest salt concentrations in drilling mud are released to groundwater immediately.

Assuming an unrealistic constant source chloride concentration of 10,000 ppm in the drilling mud, a hydraulic conductivity of $3X10^{-5}$ meters/sec (coarse sand [7]), a hydraulic gradient of 0.002 foot per foot, and an effective porosity of 0.23, we can calculate the concentration of chloride due to salt from drilling mud at a distance of 50 meters from the drill hole and after a year of transport to be less than 1 ppm.

The above equation only gives the concentration of salt as a function of time and distance from the source and illustrates the influence of processes leading to dilution. Since it is not possible to estimate the amount of time that the drilling mud will be a source of salts, except to say that it will be long compared to groundwater flow velocities, another way to estimate the hazards presented by the salts to health and the environment is to quantify the amount of water that will be necessary to dilute the salts to drinking water concentrations.

From a mass balance point of view, the amount of groundwater necessary to dilute produced water drilling mud to drinking water standards is given by the following equation:

$$V_{mud}C_{mud} + V_{gw}C_{gw} = V_{total}C_{total} \tag{4}$$

Where:

V_{mud} = volume of mud
V_{gw} = volume of groundwater needed to dilute the mud to drinking water standards
V_{total} = the total volume of water after dilution
C_{mud} = Concentration of salt in the mud
C_{gw} = Concentration of salt in the groundwater for dilution
C_{total} = Concentration of salt after dilution.

In the case of drinking water, the Federal Standard is 250 ppm chloride.

Assuming that all strata of each formation that is drilled, are penetrated by drilling mud filtrate to a diameter of 180 inches in a cylinder around the drill hole (an unrealistically conservative assumption, actual penetration averages much less: see representative examples of mud filtrate penetration in the invasion profile log located in Appendix A), a uniform mud filtrate concentration of 10,000 ppm chloride to the full radius of 90 inches, an ambient groundwater concentration of 50 ppm chloride, and an effective porosity of 0.23, we can calculate that there is enough groundwater within a radius of 52 feet from each drill hole to bring the total concentration down to 250 ppm chloride.

Examination of a typical invasion profile log (Appendix A) reveals that regions of the borehole where mud filtrate invasions equal or exceed 180 inches in diameter are rare, while regions where mud filtrate invasion is less than 90 inches in diameter are common. Furthermore, mud filtrate chloride concentrations are not uniformly 10,000 ppm chloride. Chloride concentrations vary approximately linearly from the concentration in drilling mud to the concentration in the formation (see representative examples of mud filtrate penetration in the invasion profile log located in Appendix A). A better estimate of the chloride concentration would be the volumetric average concentration (see Calculation of Volumetric Average Chloride Concentration (Appendix B)) of approximately 3670 ppm. Therefore, a more realistic mass balance calculation would use a mud filtrate penetration cylinder with a diameter of 90 inches, and yield a total concentration of 250 ppm chloride within a radius of 15 feet from each drill hole.

CONCLUSIONS

The following conclusions regarding the use of produced water containing high concentrations of dissolved salt in drilling mud can be made based on discussions above:

- Good quality water occurs only locally in the San Juan Basin.
- When produced water is used to build drilling mud, osmotic pressure acts to stabilize the mud /groundwater interface.
- Chemical interactions further stabilize the mud/groundwater interface.
- The combination of advection and dilution will reduce a chloride concentration of 10,000 ppm to less than one ppm in a year's time, over a distance of 50 meters.
- There is enough water within approximately 15 feet of the drill hole to dilute a concentration of 10,000 ppm chloride down to the Federal Drinking Water Standard of 250 ppm.

The above conclusions, taken together, indicate that it is unlikely that the use of produced water in drilling mud will negatively impact human health or the environment.

REFERENCES CITED

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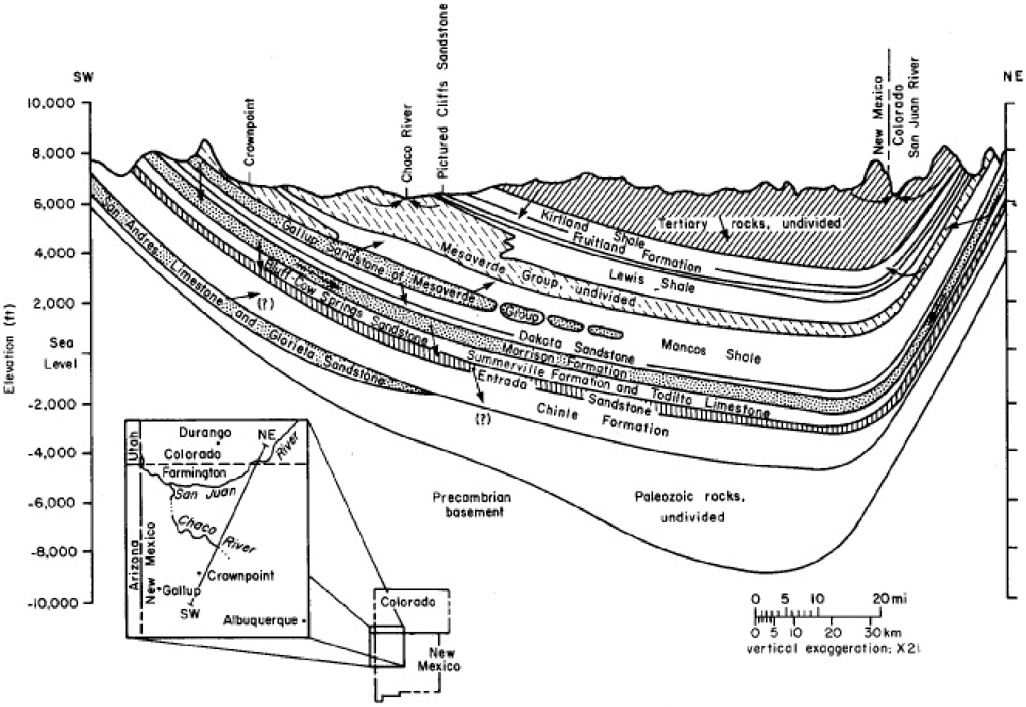


Figure 1. Generalized Hydrogeologic Cross Section of San Juan Basin, showing major aquifers (stippled), confining beds (blank), and directions of groundwater flows (arrows) (from Stone et al., 1983).

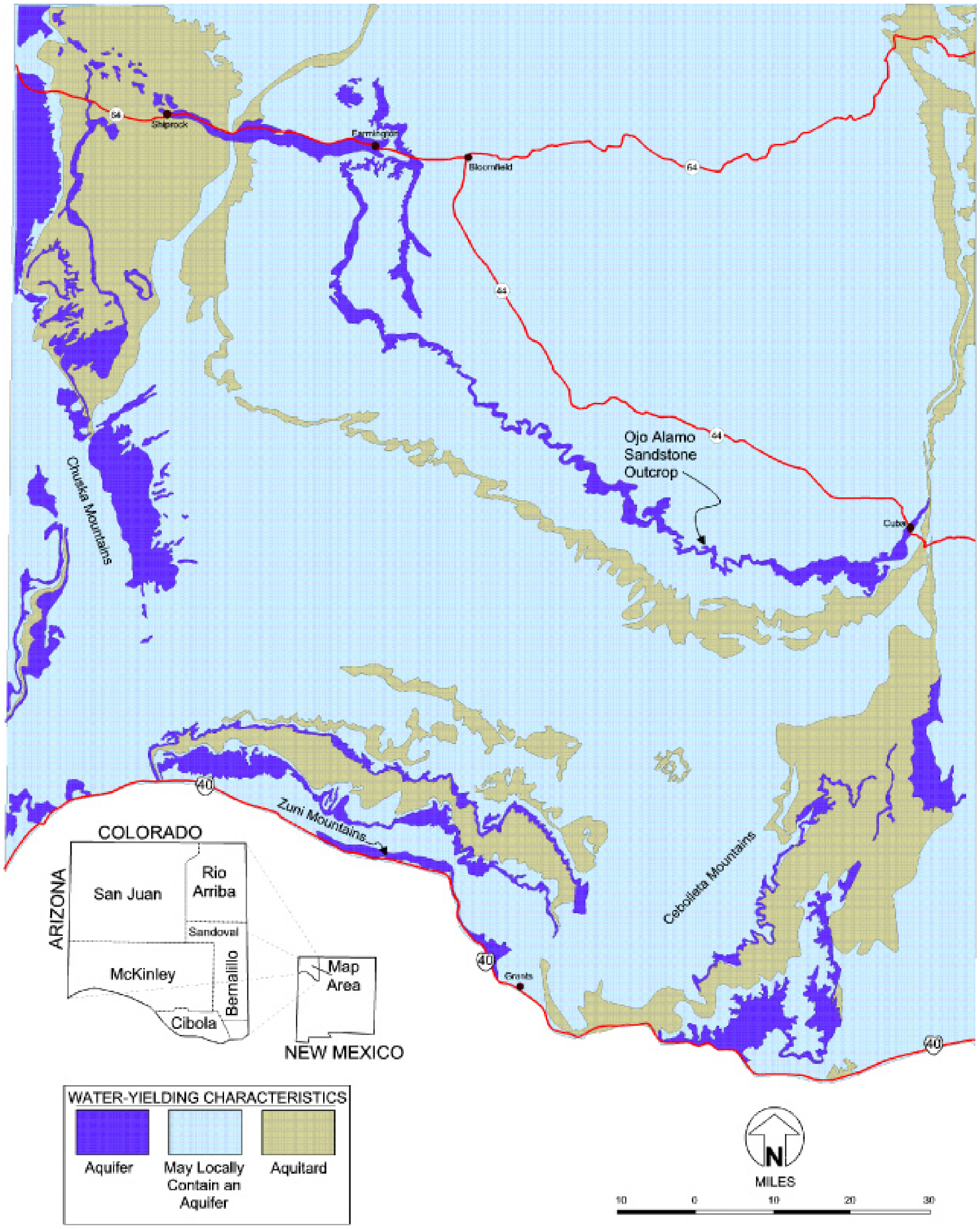


Figure 2. Hydrogeologic Map of the San Juan Basin (simplified from Stone et al., 1983)

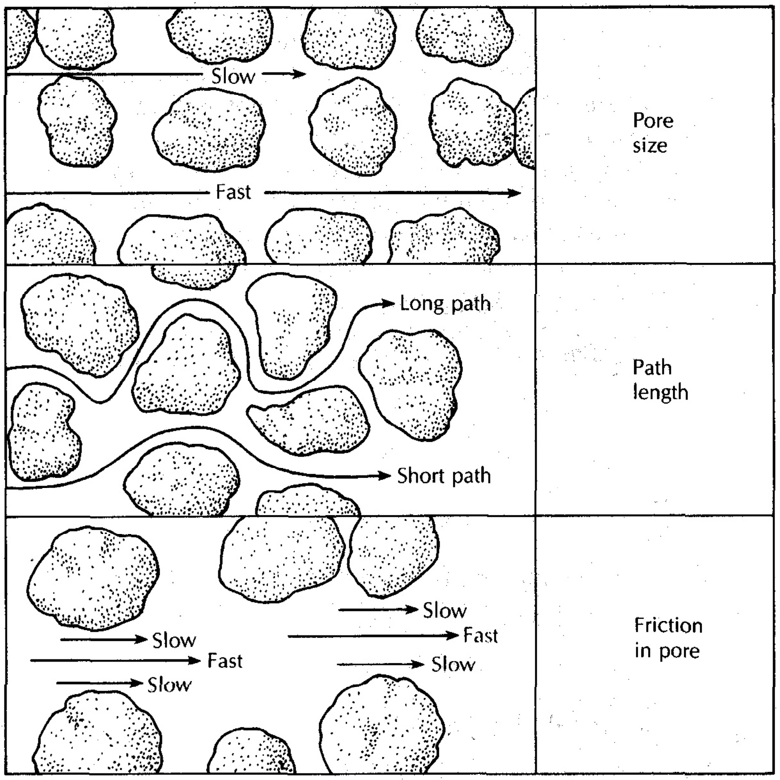


Figure 3. Factors Causing Longitudinal Dispersion (from Fetter 1988).

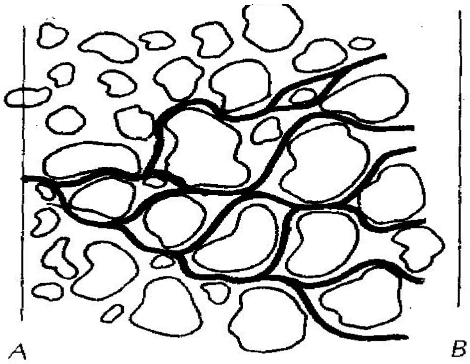
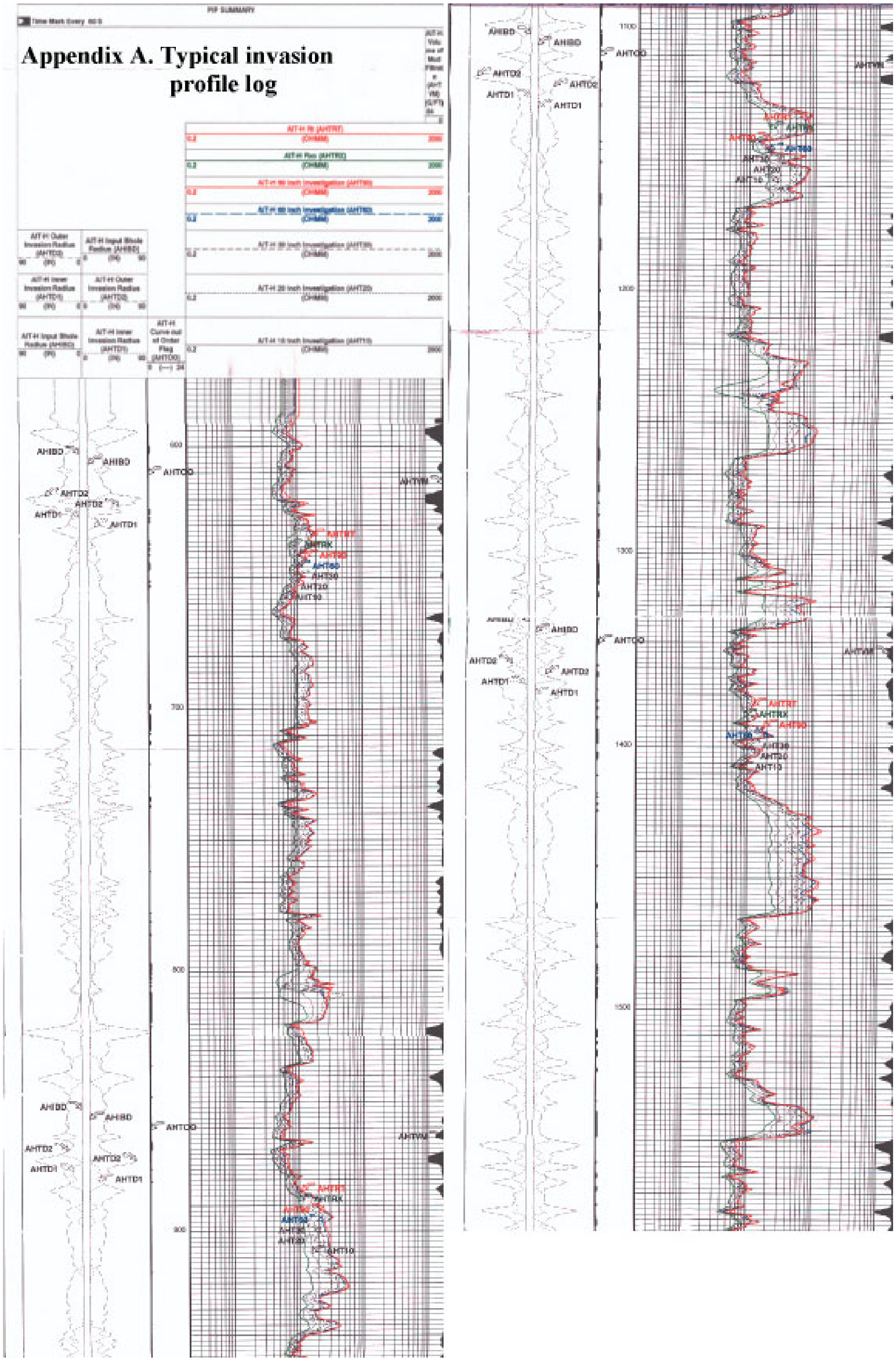
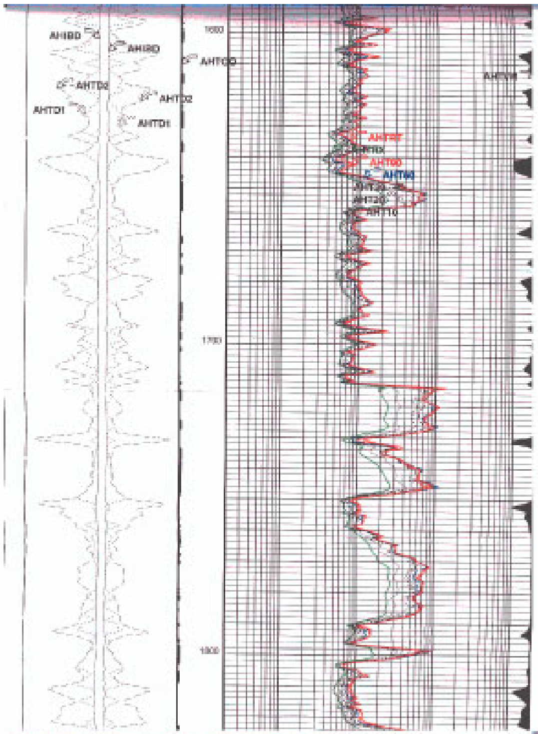
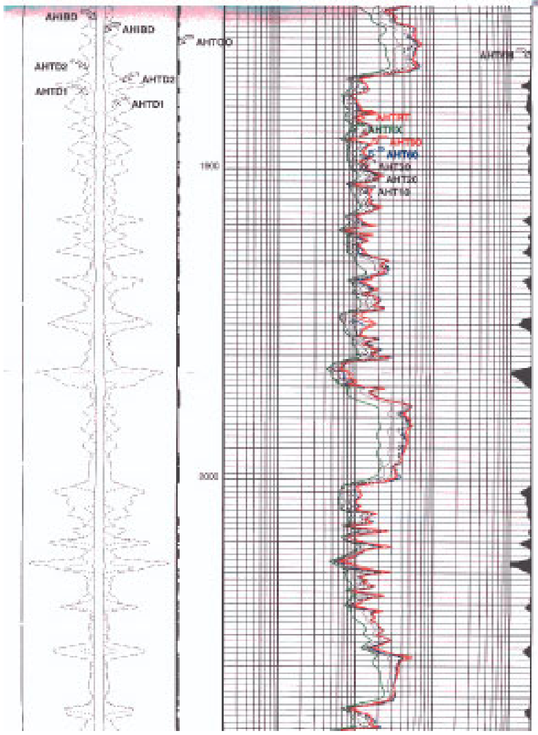


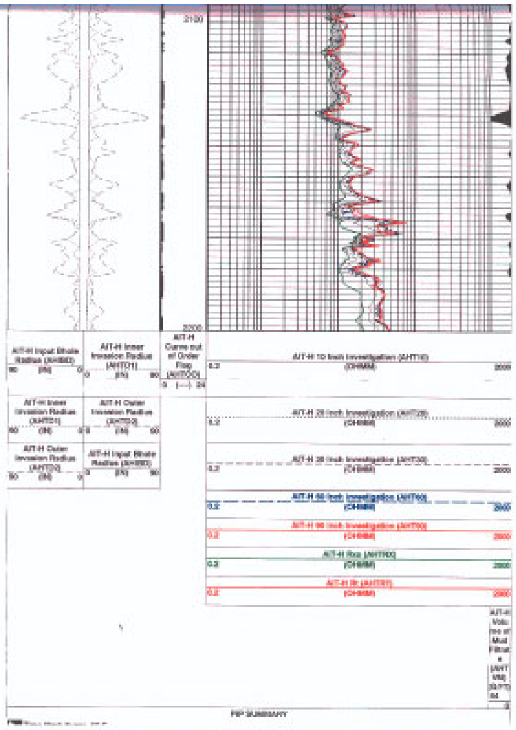
Figure 4. Flow paths in a porous medium, which cause lateral hydrodynamic dispersion (from Fetter 1988).

Appendix A. Typical invasion profile log









Tool is run in ECCENTERED mode with a tool stand-off of 0.50 IN. Bit Size is: 7.88 IN.

***** Input Selections to AIT-H Answer Product Processing *****

Caliper (GCSE): HCAL   Mud Resistivity (GRSE): AHMF   Temperature (GTSE): HTEM   Porosity (TPHI): DPHZ

***** Other Parameters used by AIT-H Answer Product Processing *****

Form Factor Exponent (FEXP)	2.000	Form Factor Numerator (FNUM)	1.000
Mud Filtrate Sample Resistivity (RMFS)	0.750 OHMM	Mud Filtrate Sample Temperature (MFST)	80.000 DEGF
Resistivity Connate Water (RW)	1.000 OHMM		

***** AIT-H Answer Product Processing Control Parameters *****

Playback Mode: NORMAL

Parameters

DLIS Name	Description	Value	
AHBHM	Array Induction Borehole Correction Mode	2_ComputeStandoff	
AHBHV	Array Induction Borehole Correction Code Version Number	890	
AHBLM	Array Induction Basic Logs Mode	8_One_Two_and_Four	
AHBLV	Array Induction Basic Logs Code Version Number	108	
AHCDE	Array Induction Casing Detection Enable	Yes	
AHCEN	Array Induction Tool Centering Flag (in Borehole)	Eccentered	
AHCSED	Array Induction Casing Shoe Estimated Depth	-50000	FT
AHFRSV	Array Induction Response Set Version for Four ft Resolution	40.70.24.21	
AHMRF	Array Induction Mud Resistivity Factor	1	
AHORSV	Array Induction Response Set Version for One ft Resolution	40.70.24.21	
AHRPV	Array Induction Radial Profiling Code Version Number	708	
AHSPV	Array Induction Radial Parametrization Code Version Number	223	
AHSTA	Array Induction Tool Standoff	0.5	IN
AHTRSV	Array Induction Response Set Version for Two ft Resolution	40.70.24.21	
BHT	Bottom Hole Temperature (used in calculations)	119	DEGF
BS	Bit Size	7.875	IN
DFD	Drilling Fluid Density	9.20	LB/G
DO	Depth Offset for Playback	0.0	FT
FEXP	Form Factor Exponent	2	
FNUM	Form Factor Numerator	1	
GCSE	Generalized Caliper Selection	HCAL	
GDEV	Average Angular Deviation of Borehole from Normal	0	DEG
GGRD	Geothermal Gradient	0.01	DF/F
GRSE	Generalized Mud Resistivity Selection	AITH_RESEST	
GTSE	Generalized Temperature Selection	HSTE_HTEM	
MST	Mud Sample Temperature	68.00	DEGF
PP	Playback Processing	NORMAL	
RMFS	Resistivity of Mud Filtrate Sample	0.7500	OHMM
SHT	Surface Hole Temperature	75	DEGF
TD	Total Depth	7706	FT

Format: AITH_Resistivity/ImageTen   Vertical Scale: 5" per 100'   Graphics File Created: 12-Jun-2001 15:50

OP System Version: 9C2-303

MCM

HILTB-CTS   OP92-KP1

Input DLIS Files

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Output DLIS Files

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APPENDIX B
CALCULATION OF THE VOLUMETRIC AVERAGE CHLORIDE CONCENTRATION

The mud filtrate invades a cone around the well bore. The chloride concentration of the mud filtrate is assumed to vary linearly from the injected mud concentration to the chloride concentration of groundwater in the formation. The average concentration can be calculated geometrically by envisioning a vertical tubular shape with the base defined by the area of an invasion cylinder around the well bore and the height defined by the chloride concentration (Figure B1). The volume of the shape divided by the area of the base gives the volumetric average concentration of chloride.

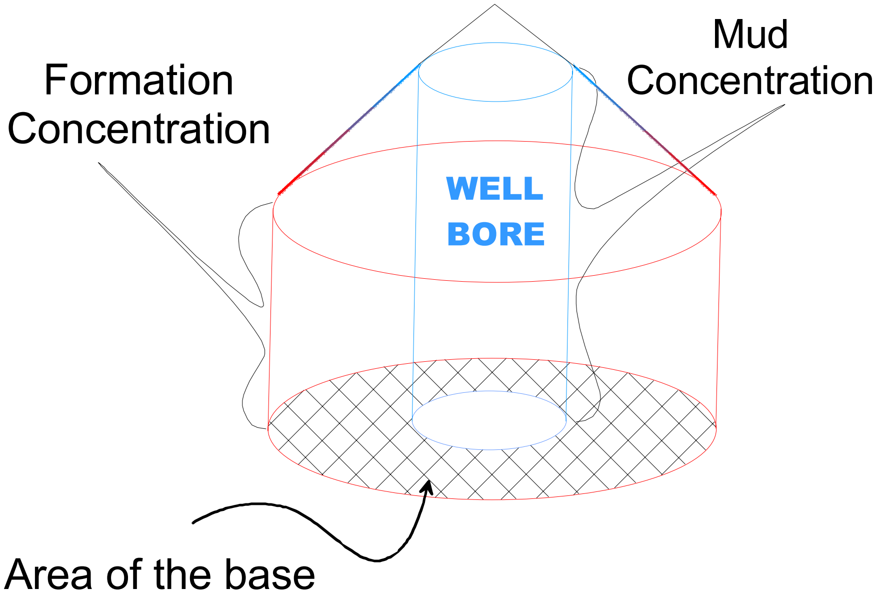

Figure B 1. Shape with the base defined by the area of an invasion cylinder around the well bore and the height defined by the chloride concentration

The volume of the shape is composed of the invasion cylinder (**A**) , plus the truncated cone on top of it (**B**), minus the small cone on top(**C**), and the volume of the well bore (**D**) (Figure B2).

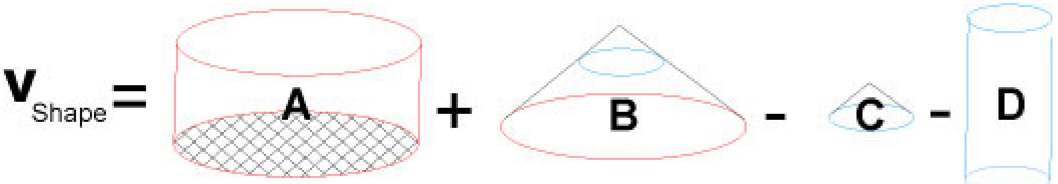

Figure B 2. Visual formula for the volume of the shape defining the volumetric chloride concentration. **A** = invasion cylinder, **B** = truncated cone, **C** = small cone on top, **D** = well bore.

The volume of a cylinder can be calculated as follows:

$$\text{Volume Cylinder} = (\text{Pi})*(\text{radius}(r))^2*(\text{height}(h)) \tag{1}$$

The volume of a cone can be calculated as follows:

$$\text{Volume Cone} = 1/3(\text{Pi})*(\text{radius}(r))^2*(\text{height}(h)) \tag{2}$$

To calculate the volumetric average concentration of the shape we define a number of variables shown in Figure B3.

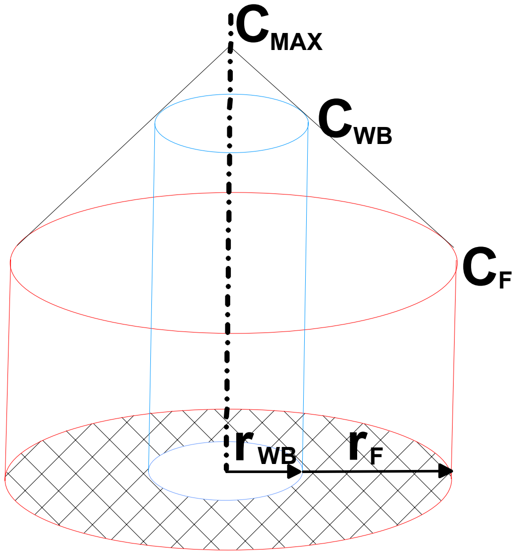

Figure B 3. Definition of variables in calculation of volumetric chloride concentration, showing radius of the well bore ($\mathbf{r_{WB}}$), radius of invasion into the formation ($\mathbf{r_F}$), chloride concentration in the well bore ($\mathbf{C_{WB}}$), chloride concentration in the formation ($\mathbf{C_F}$), and the chloride concentration at the top of the cone ($\mathbf{C_{MAX}}$)

$\mathbf{C_{MAX}}$ can be defined in terms of the variables introduced in Figure B3 using similar triangles (Figure B4) as follows:

$$\mathbf{C_{MAX}}=\mathbf{C_F}+((\mathbf{C_{WB}}-\mathbf{C_F})*(\mathbf{r_F}/(\mathbf{r_F}-\mathbf{r_{WB}}))) \tag{3}$$

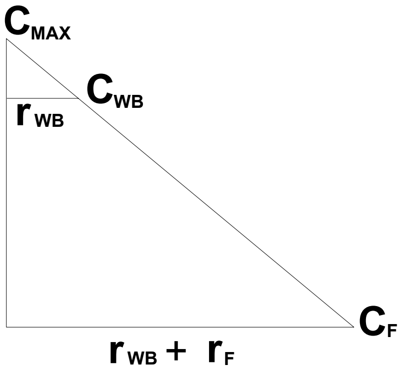

Figure B 4. Basis for defining C_{MAX} in terms of variables introduced in Figure B3.
Shape components from Figure 2 are calculated as follows:

$$\mathbf{A} = \mathrm{Pi}*\mathbf{r_F}^{2}*\mathbf{C_F} \tag{4}$$

$$\mathbf{B} = 1/3\ \mathbf{Pi}*\mathbf{r_F}^{2}*(\mathbf{C_{MAX}} - \mathbf{C_F}) \tag{5}$$

$$\mathbf{C} = 1/3\ \mathbf{Pi}*\mathbf{r_{WB}}^{2}*(\mathbf{C_{MAX}} - \mathbf{C_{WB}}) \tag{6}$$

$$\mathbf{D} = \mathrm{Pi}*\mathbf{r_{WB}}^{2}*\mathbf{C_{WB}} \tag{7}$$

The area of the base of the shape used to calculated as follows:

$$\mathbf{AREA} = (\mathrm{Pi}*\mathbf{r_F}^{2}) - (\mathrm{Pi}*\mathbf{r_{WB}}^{2}) \tag{8}$$

Finally, the volumetric average chloride concentration ($\mathbf{C}_{AVE}$) can be calculated as follows:

$$\mathbf{C_{AVE}} = \mathbf{(A+B-C-D)/AREA} \tag{9}$$

A series of volumetric average concentrations for invasion cylinders of increasing radius is shown in Figure B5.

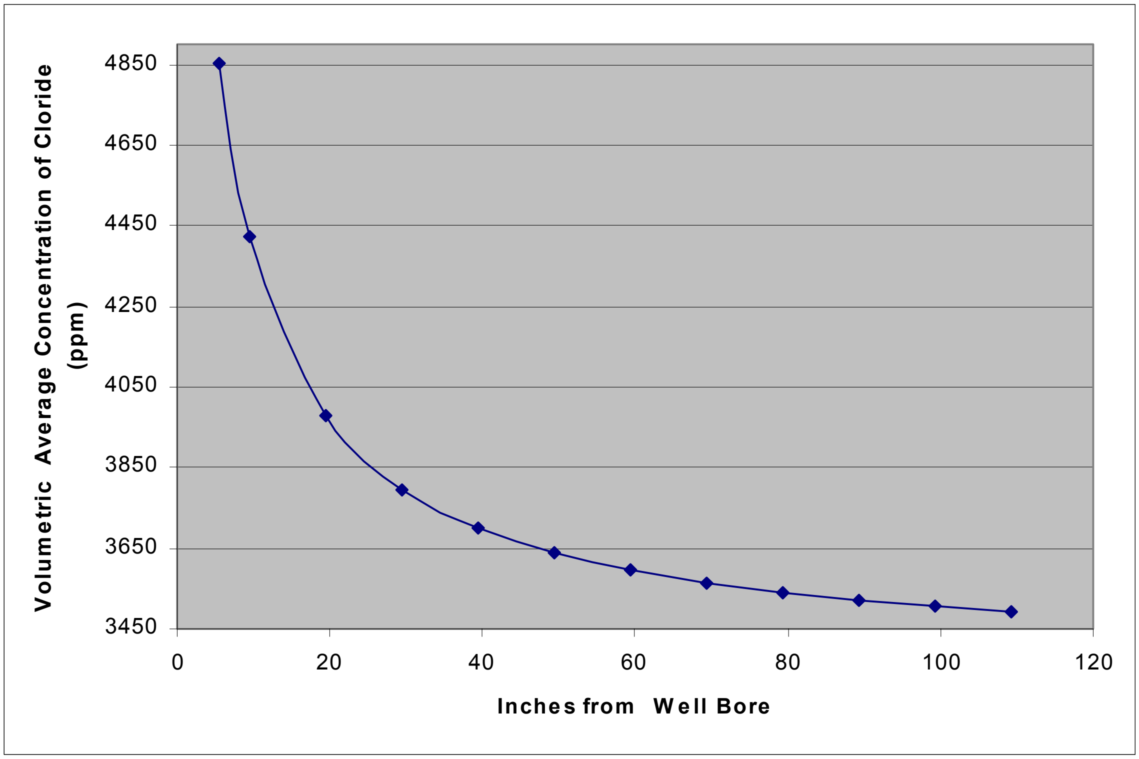

Figure B 5. A series of volumetric average concentrations for invasion cylinders of increasing radius